Astronomical Measurements in Ancient Greece

General Lyceum of Nea Zichni
Spring 2013

Aim

Combining observations of the Earth, the Moon and the Sun with geometry and trigonometry knowledge, we will repeat the measurements carried out by Ancient Greek philosophers in order to estimate the radius of the Earth, the Moon and the Sun and the distances between the Earth and the Moon, and the Earth and the Sun.

Introduction

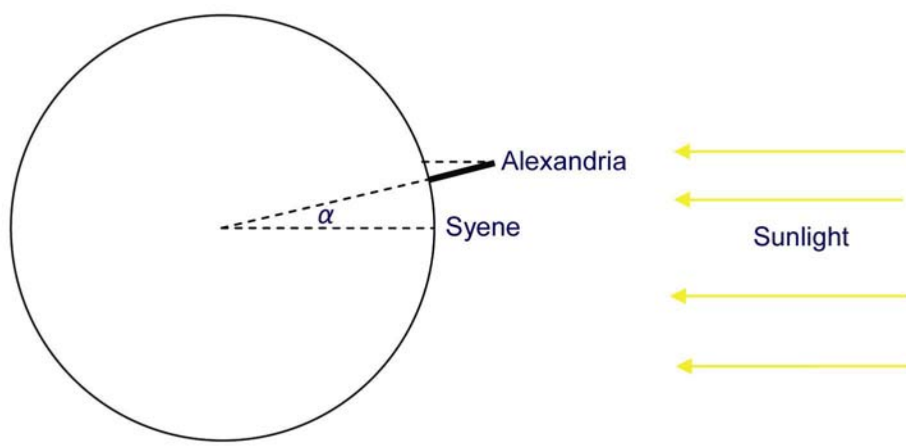
Casual sky observations reveal that heavenly bodies trace the same paths year after year. But only these observations do not suffice to estimate the sizes or the distances of these objects. In fact, up to 150 years ago we were not even aware of the true size of the Universe in which we live. Modern observations which estimate the diameter of the Universe at approximately 28 billion light years are based on older measurements of smaller distance scales (distances of far away galaxies), which are based on older measurements of even smaller distances (distances of nearby galaxies), which rely on smaller distance measurements (distances of far away and nearby stars), which were made possible only after distance measurements of our own solar system objects were carried out. Therefore, our cosmic distance scale ladder was initially founded on the pioneering efforts of the people who first were able to measure the distances between the Earth and its closest massive object, the Moon, and the Earth and its closest star, the Sun.
These measurements were first carried out about 2500 years ago, by Ancient Greek philosophers, like Eratosthenes from Cyrene (276 – 196 B.C.) and Aristarchus from Samos (310 – 230 B.C. approx.). Eratosthenes held the view that the Earth was spherical long before Galileo made a similar proposal and with simple observations was able to accurately measure the radius of the Earth. Aristarchus suggested, long before Copernicus put forward his model of our solar system, that the Sun was at the center of the known Universe and that the Earth and the planets revolved around the Sun, while the stars are at huge distances from the Sun. Using this model he was able to measure the sizes of the Moon and the Sun and estimate their distances from the Earth.
And all these measurements in Ancient Greece were made without the use of telescopes or other similar instruments, which allow for accurate observations. Moreover, the observations of the time did not allow for easy verification and acceptance of the models proposed by Eratosthenes and Aristarchus, so most people at the time held other views for the Universe. We will repeat the measurements carried out by these pioneering philosophers, demonstrating that the methods they suggested are correct and can yield very accurate results when combined with modern techniques.

The Earth's radius

This measurement was carried out by Eratosthenes using a very simple model for the Earth and the Sun, according to which:

a) the Earth is spherical, as can be deduced from observations of the Earth's shadow on the Moon during eclipses of the Moon and by the way a ship setting out from a port towards the open sea disappears in the horizon, and
b) the Sun is at infinitely large distance form the Earth so that its rays arrive at the Earth parallel to each other.

Eratosthenes knew that at noon on one particular day (summer solstice) in the town of Syene in Egypt (present-day Aswan) the Sun is reflected in the bottom of a deep vertical well without casting any shadows on the water from the sides. Therefore, at that time the Sun lies exactly at the zenith, directly over the town. But this never happened in Alexandria, were Eratosthenes lived, and even at that time all objects in Alexandria cast shadows. This cannot happen if the Earth is flat and the Sun is very far away and Eratosthenes realized that the phenomenon was a direct consequence of the Earth's curvature and could be used to measure the Earth's radius. Measuring the length of the shadow of a vertical stick at Alexandria at noon on summer solstice he was able to deduce that the sun rays were meeting the Earth with an angle of α=7,2 degrees (one fiftieth of a complete circle).



He then used the relation:

$$\frac{\text{Distance Syene-Alexandria}}{\text{Circumference of the Earth}} = \frac{7,2^{o}}{360^{o}}$$

and was able to estimate the circumference of the Earth, after having the distance between Alexandria and Syene measured and found about 805 km.
From the above what did Eratosthenes found for the circumference and the radius of the Earth?

(measured values appear in red, bibliography values appear in blue)

Circumference of the Earth: 40250 km

Radius of the Earth: 6406 km

Which are todays accepted values?

Circumference of the Earth: 40074 km

Radius of the Earth: 6378 km

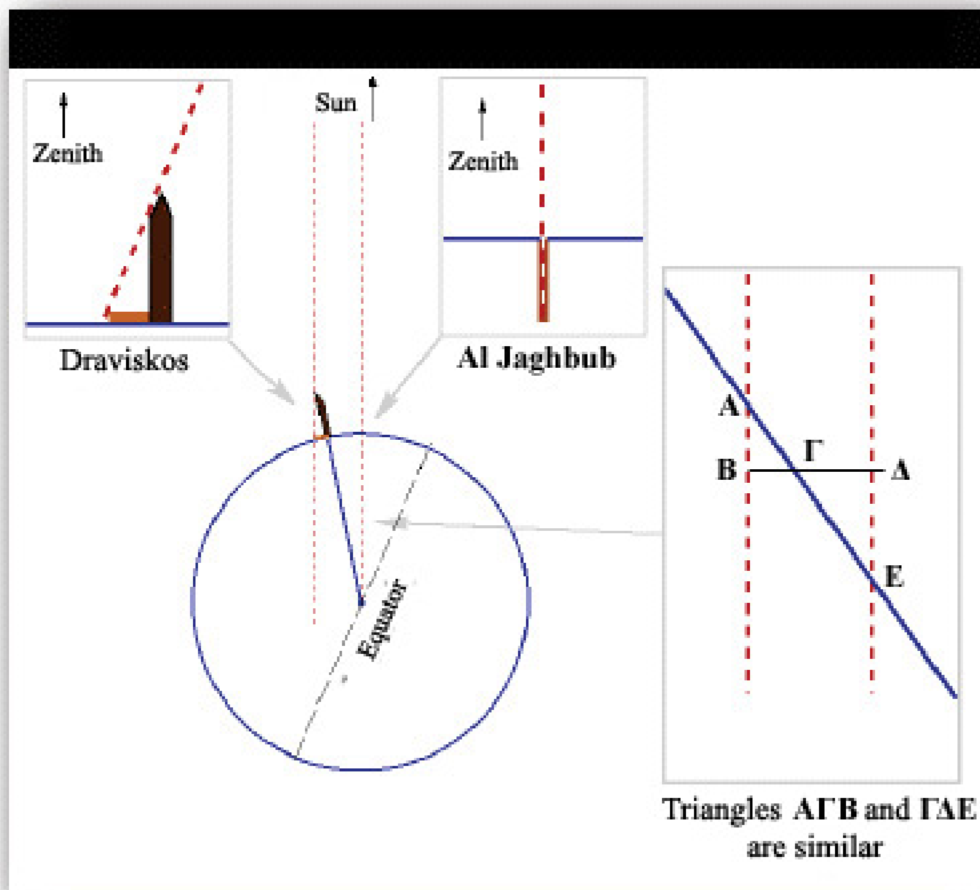
Measurement activities

In the Sahara desert, in Al Jaghbub there is a deep vertical well with water, which has been observed every year at noon on the summer solstice not to cast any shadows in the water from its sides. On the same day you happen to be at the village of Draviskos, nearby Nea Zichni, and at noon, exactly when the sun is directly over Al Jaghbub, you take a picture of a tree with its shadow, in order to repeat Eratosthenes measurements.

From the similar triangles ΑΒΓ and ΓΔΕ we see that

$$\frac{\text{Tree height}}{\text{Tree shadow length}} = \frac{\text{Earth radius}}{\text{Distance Draviskos-Al Jaghbub}}$$

as can be verified from the picture below.



Using a ruler measure the height of the tree and the length of its shadow from the above photograph.

Height of tree: 7 cm

Length of tree's shadow: 2 cm

In order to estimate the distance between Draviskos and Al Jaghbub you can use a map from the Internet, like the one shown below. In this map measure the distance between the two places in cm and then convert it to km, using the scale shown on the map.

Distance between Draviskos and Al Jaghbub on the map in cm: 13.4 cm

Length in cm of 500 km according to the map's scale: 3.5 cm

Distance between Draviskos and Al Jaghbub in a straight line in km: 1914 km

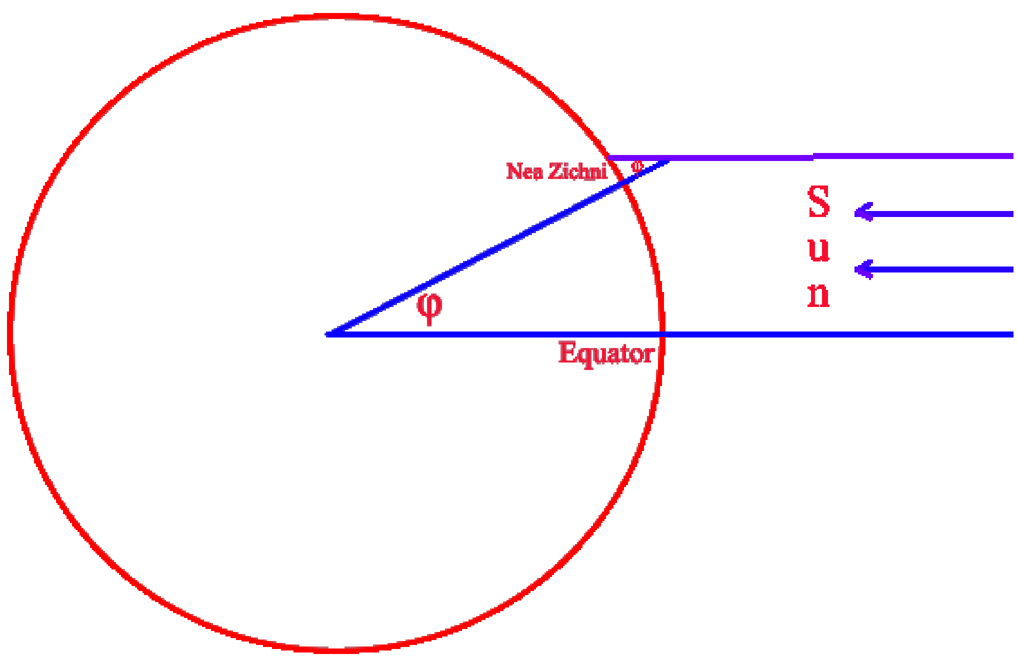
Using the measurements you made above estimate the radius of the Earth:

Radius of the Earth: 6699 km

What's the error in your estimate compared to the accepted value?

Error of estimate: 5%

Another way to do the same measurement is by utilizing the observation that on the day of spring equinox the length of the day equals the length of the night. This happens because at noon of this day the Sun is directly over the equator. Then, measuring the angle at which the sun rays fall on Nea Zichni at that time, is a direct measurement of Nea Zichni's geographic latitude, as can be seen on the graph below.



At noon on spring equinox (13.00 p.m. local time on 20th March 2013) we measure the height of a metal rod and then we place it vertical in the schoolyard and measure its shadow. Write down your measurements:

Height of the metal rod: 79.2 cm
Length of its shadow: 68.9 cm

By using trigonometry (or constructing an orthogonal triangle and measuring the angles on the construction) we find that the angle with which the sun rays meet the ground in Nea Zichni and hence the geographic latitude of Nea Zichni. Write down your finding:

φ = 41.022 degrees

From software like Google Earth we can find that the latitude for the Lyceum of Nea Zichni. What is it?
Nea Zichni latitude: 41.027 degrees
What is then the error in the estimate of the geographic latitude of Nea Zichni?
Error: 0.01%

From this result we can then estimate the circumference of the Earth since

$$\frac{\text{Distance Nea Zichni-Equator}}{\text{Circumference of the Earth}} = \frac{41,022^{o}}{360^{o}}$$

We use Google Earth to measure the distance from Nea Zichni to the equator. We find:

Distance of Nea Zichni to equator: 4543.6 km

Circumference of the Earth: 39874 km

Radius of the Earth: 6346 km

Error of measurement: 0.5%

This is a very accurate measurement indeed!

The size and the distance of the Moon

In order to measure the size of the Moon we will follow Aristarchus and use observations of lunar eclipses. Aristarchus measured the time it took for the Moon to pass through the Earth's shadow during a total lunar eclipse and compared this time to the time it takes the Moon to move in the night sky by one full lunar diameter. He found that this ratio is about 8/3 and thus, if the Sun is infinitely far and the sun rays traveled parallel to each other when reaching the Earth, this ratio is how larger the diameter of the Earth is from the diameter of the Moon. So, using Aristarchus measurements, we find

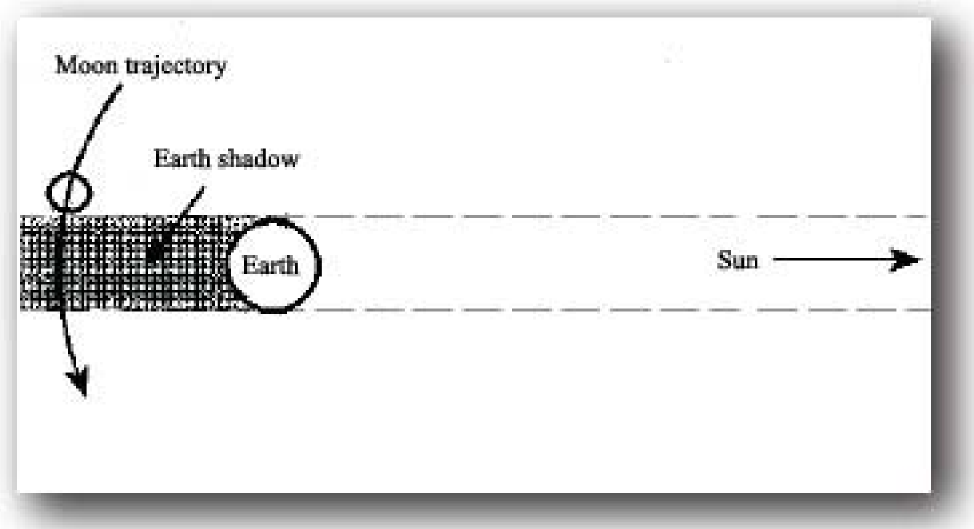
Radius of the Moon: 2380 km

What is the today accepted value?
Radius of the Moon from bibliography: 3476 km

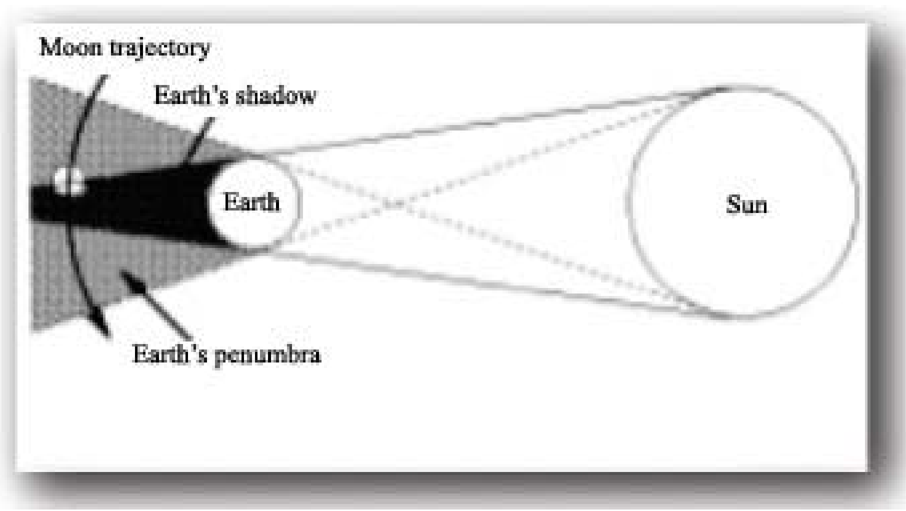
Why do you think there is a large error in this estimate?
The Sun is not infinitely far away and the sun rays cannot be though of as being parallel over such large distances as the length of the shadow of the Earth. Therefore the model used by Aristarchus needs to be corrected.

This model contains the following elements:

a) The Earth is spherical
b) the Sun is infinitely far from the Earth and the sun rays are parallel to each other when they reach the Earth
c) the Moon revolves around the Earth so that at some point it comes inside the Earth's shadow and thus lunar eclipses happen
This model is described in the graph below.

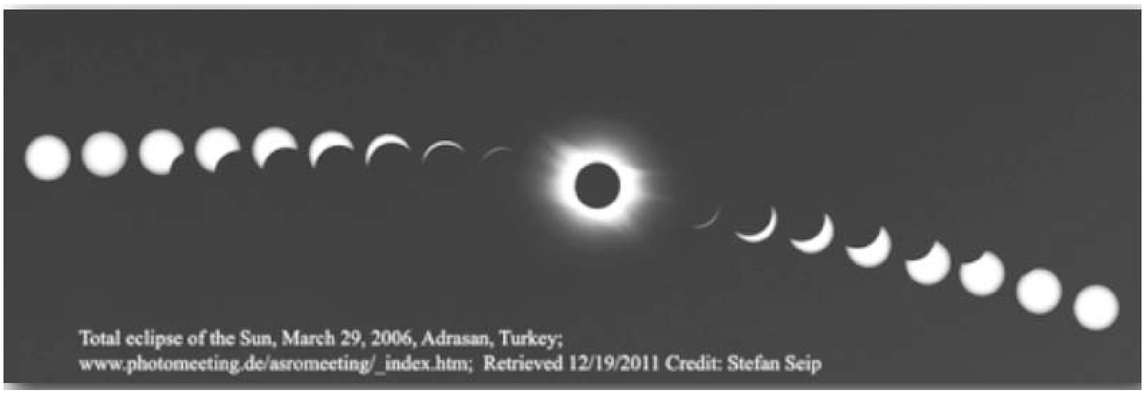


However, the Sun is not infinitely far and the sun rays are not completely parallel. A better model of a lunar eclipse is shown in the graph below, from which it can be inferred that during a lunar eclipse the Moon traverses a distance smaller than the diameter of the Earth.



In order to estimate the length of the shadow of the Earth compared to the Earth's diameter, it suffices to measure the distance of the shadow cast by a round metal coin held against the Sun at a distance such that it just hides the solar disk. Such an observation however is dangerous for the naked eyes and we therefore contact a similar observation for the Moon, since, as already noticed by Ancient Greeks, the Moon and the

Sun subtend equal angles in the sky. This can also be seen in the picture below, since the disk of the Moon is just equal to the disk of the Sun and covers it exactly in a solar eclipse.



If we then hold a 2 cent coin with a diameter of 1.9 cm, in front of our eye at such a distance that the disk of the Moon matches the size of the coin we find that we have to place the coin at approximately 2 meters and 5 centimeters away (2.05 m). In other words, the ratio of the distance of the coin to our eye to the diameter of the coin is

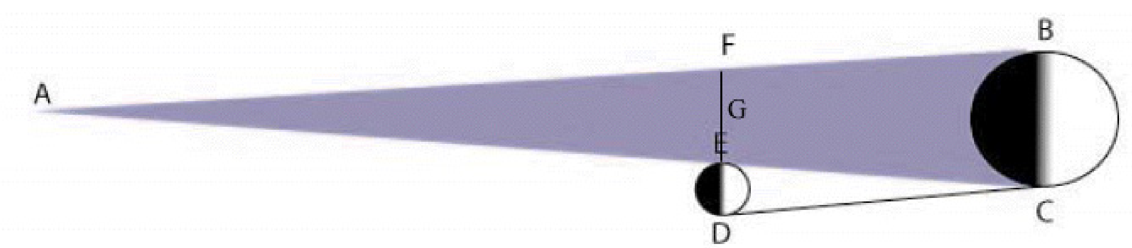
$$\frac{\text{length of coin shadow}}{\text{coin diameter}} = \frac{205 \text{ cm}}{1{,}9 \text{ cm}} \cong 108$$

It is apparent that the same holds for the shadows cast by the Earth and the Moon due to the sun rays. Therefore the length of the shadow of the Earth equals to

$$\text{Length of the shadow of the Earth} = 108 \times \text{ Diameter of the Earth}$$

But the Earth is almost at the tip of the Moon's shadow, as can be seen during solar eclipses and therefore the length of the shadow of the Moon is equal to the mean distance between the Earth and the Moon. Thus:

$$\frac{\text{length of shadow of the Moon}}{\text{diameter of the Moon}} = \frac{\text{distance Earth-Moon}}{\text{diameter of the Moon}} \cong 108$$



As can be seen from the above graph, where the isosceles triangles AFE and ABC are similar, the ratio of AG to FE is also the same. Thus

$$\frac{AG}{FE} \cong 108$$

and since the length of the shadow of the Earth is equal to the length of AG plus the distance between the Earth and the Moon, we find that:

Length of Earth's shadow = AG + distance Earth-Moon = 108xFE + 108xMoon diameter

or

108 x Earth diameter = 108 x (FE + Moon diameter)

or

Diameter of the Earth = FE + Diameter of the Moon

Utilizing then the measurement carried out by Aristarchus and a more realistic model we can estimate quite accurately the diameter of the Moon. In order to measure ourselves the length FE we can use a photograph of a lunar eclipse, like the one below. In this we need to estimate how large is the shadow of the Earth falling on the Moon compared to the Moon's diameter. This is the length FE in Moon diameters.

How long (in cm) is the diameter of the Moon in this photograph?

Diameter of the Moon: 8.2 cm

Draw a curve at the edge of the shadow of the Earth on the Moon. Then draw the circle corresponding this arc (you can find the center of this circle by using two arc segments and finding the point of intersection of the perpendicular lines passing through their centers). Which is the diameter of this circle (in cm)?

Diameter of the shadow of the Earth: 20.5 cm

How many times longer is the diameter of the shadow of the Earth compared to the diameter of the Moon?

Length of FE (in Moon diameters): 2.5

Using the value of the radius of the Earth you found earlier, how long is the diameter of the Moon?

Diameter of the Moon: 3626 km
Radius of the Moon: 1813 km

What is your error compared to the accepted value?

Error in measurement: 4.3%

From the above measurements the distance of the Moon can be estimated, following Aristarchus ideas. How far is the Moon from the Earth?

Distance between the Earth and the Moon: 391608 km

Which is the mean distance between the Earth and the Moon according to the bibliography?

Mean Earth-Moon distance (bibliography): 384400 km

What is the error in your measurement?

Error in measurement: 1.9%

The size and the distance of the Sun

Aristarchus also found a way to estimate the distance of the Sun. His method relies on the observation that when the Moon is at first or last quarter, the angle between the Sun, the Moon and the Earth is a right angle as shown in the figure below.

Therefore the model used contains the following elements:
a) the Earth is spherical
b) the Sun is at a great distance from the Earth but not infinitely large, so that the sun rays are not parallel to each other when they reach the Earth
c) the Moon revolves around the Earth

By measuring the angle Sun-Earth-Moon when the Moon is at the phase of the first or last quarter and knowing the distance of the Moon, we can estimate the distance of the Sun. However, this angle is very close to 90 degrees and any offset is very hard to measure. Moreover it is also difficult to find the exact time when the Moon is at the first or the last quarter, as can be realized from the pictures below.

the first quarter

the last quarter

Aristarchus himself measured this angle to be about 87 degrees. This measurement has a large error. Later measurements by other Greek philosophers found this angle to approximately 89.5 degrees, while modern more accurate measurements yield a value of 89.853 degrees. Therefore, according to the above graph

$$\cos\theta = \frac{\text{Distance Earth-Moon}}{\text{Distance Earth-Sun}}$$

and thus, using the distance of the Moon estimated above and the modern measurements of θ, how long is the Sun away from the Earth?

Distance between the Earth and the Sun: 152635621 km

You have just measured the length of the Astronomical Unit!

What is the value accepted today?
Distance between Earth and Sun (bibliography): 149600000 km

What is the error in your estimate? 2%

We know, as stated above, that the angle subtended by the Sun as seen from the Earth is the same as the angle subtended by the Moon as seen by the Earth. Therefore, the ratio of the distance Earth-Sun to the diameter of the Sun is the same as the ratio of the distance Earth-Moon to the diameter of the Moon:

$$\frac{\text{distance Earth-Sun}}{\text{diameter of the Sun}} = \frac{\text{distance Earth-Moon}}{\text{diameter of the Moon}} \cong 108$$

From the above relation we can calculate the diameter and the radius of the Sun. We find that:

Diameter of the Sun: 1413292 km
Radius of the Sun: 706646 km

You have just measured the radius of a star!

What is the currently accepted value?

Radius of the Sun (bibliography): 696600 km

What's your error? 1.5%

Bibliography

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